Towards Lower Bit Multiplication for Convolutional Neural Network Training

Kai Zhong [1] Tianchen Zhao [2] Xuefei Ning [1] Shulin Zeng [1] Kaiyuan Guo [1] Yu Wang [1] Huazhong Yang [1]

Abstract

Convolutional Neural Networks (CNNs) have been widely used in many fields. However, the training process costs much energy and time, in which the convolution operations consume the major part. In this paper, we propose a fixed-point training framework, in order to reduce the data bit-width for the convolution multiplications. Firstly, we propose two constrained group-wise scaling methods that can be implemented with low hardware cost. Secondly, to overcome the challenge of trading off overflow and rounding error, a shiftable fixed-point data format is used in this framework. Finally, we propose a double width deployment technique to boost inference performance with the same bit-width hardware multiplier. The experimental results show that the input data of convolution in the training process can be quantized to 2-bit for CIFAR-10 dataset, 6-bit for ImageNet dataset, with negligible accuracy degradation. Furthermore, our fixed-point training framework has the potential to save at least 75% energy of the computation in the training process.

1. Introduction

Deep convolutional neural networks (CNNs) have achieved state-of-the-art performance in many computer vision tasks, such as image classification (Krizhevsky et al., 2012; He et al., 2016) and object detection (Redmon et al., 2016; Liu et al., 2016). However, deep CNNs are computation and storage-intensive. The training process could consume up to hundreds of ExaFLOPs of computations and tens of GBytes of memory storage, thus posing a tremendous challenge for training in resource-constrained environments. At present, the most common training method is to use GPUs, but it consumes much energy. The power of one running GPU is about 250W, and it usually takes more than 10 GPU-days to train one CNN model on commonly used ImageNet(Deng et al., 2009) dataset. It makes the application of AI algorithm expensive and not environmental friendly.

Reducing the precision of neural networks has drawn great attention due to its potential in significantly reducing both the memory and computational complexity in order to reduce the energy consumption. It has been pointed out that the power consumption and circuit area of fixed-point multiplication and addition units are greatly reduced compared with floating-point ones (Sze et al., 2017). Many studies (Jacob et al., 2017a) focus on amending the training process to acquire a reduced precision model with a better inference performance. Dong et al. (2019); Banner et al. (2018b) are capable of lowering the bit-width down to 4 while retaining the performance. However, they usually rely on tuning from a full-precision pre-trained model, which already costs a lot, or even introduce more operations into training for a better inference performance. Other methods (Zhou et al., 2016; Rastegari et al., 2016) with an extremely low bit-width (e.g., 1 or 2) struggle in bigger datasets, such as ImageNet, and cause non-negligible accuracy degradation.

Table 1. The number of different operations in the training process when the batch size is one. “EW-Add” is the abbreviation for element-wise-add, ”F” and ”B” indicate the forward and backward processes respectively.

Op Name	Op Type	Res18	Res20
Conv (F)	Mul&Add	2.72E+10	4.05E+07
Conv (B)	Mul&Add	5.44E+10	8.11E+07
BN (F)	Mul&Add	3.01E+07	1.88E+05
BN (B)	Mul&Add	3.01E+07	1.88E+05
EW-Add (F)	Add	1.49E+07	7.37E+04
EW-Add (B)	Add	1.20E+07	7.37E+04
Params Update (B)	Add	1.12E+07	2.68E+05

Besides the optimization for inference, there are a few studies aiming at accelerating the training process. Wang et al. (2018); Mellempudi et al. (2019) reduces the bit-width of floating points to 8 during the training process to save the computation effort and than apply different format in forward and backward process to get better accuracy. Wu et al. (2018) implements a full-integer training procedure to save the hardware cost further but fails to get acceptable performance.

As shown in Table 1, Conv(F) and Conv(B) in the training process consume the majority of all the operations. More-

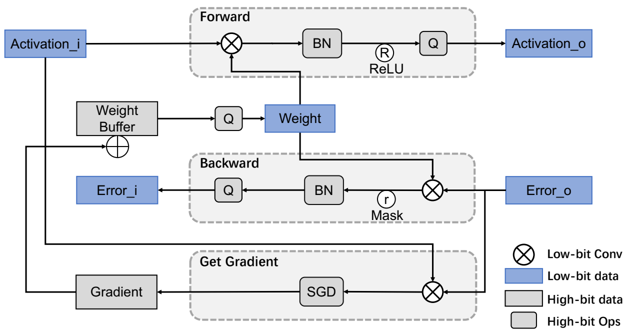

Figure 1. Computation flow of the fixed-point training. Floating-point data are quantized to low-bit before the convolution operations.

over, since the cost of multiplications is higher than additions (Sze et al., 2017), the multiplications in convolution account for the main computational energy consumption of the CNN training process. Therefore, this work aims at simplifying multiplications in convolution to low-bit fixed-point operations, while retaining a similar performance with the full-precision baseline. The contributions of this paper are:

1. We propose a shiftable data format and two constrained group-wise scaling methods to improve the representation ability of fixed-point numbers. We could improve ResNet-20 accuracy on CIFAR-10 from 60.98% to 90.63% in the training scenario with 4-bit fixed-point convolution multipliers.

2. We propose a double width deployment technique with the help of the high-precision weight buffer in order to boost the inference performance without introducing new computing elements. With this technique, ResNet-20 trained on CIFAR-10 with 2-bit could achieve better accuracy than 3-bit training without double width deployment by 1.63%, even closed to the result of 4-bit training(0.7% lower). For 4-bit ResNet-18 training on ImageNet, the accuracy could improve by 1.7%.

3. Experimental results demonstrate the effectiveness of our methods. On CIFAR-10, ResNet-20 achieves an accuracy of 90.75% (1.45% accuracy drop) with 2-bit, 91.74% (0.79% accuracy drop) with 3-bit fixed-point multiplications. On ImageNet, we train ResNet-18 to an accuracy of 68.2% with 6-bit fixed-point multiplications, which is better than that of the state-of-the-art fixed-point training (66.4% with hybird 8-bit and 16-bit quantization).

2. Related work

2.1. Post-Training Quantization

Earlier quantization methods like (Han et al., 2015) focused on the post-training quantization, and quantized the pre-trained full-precision model using the codebook generated by clustering or other criteria (e.g., SQNR (Lin et al., 2015), entropy (Park et al., 2017)). Banner et al. (2018b) selected the quantization bit-width and clipping value for each channel through the analytical investigation, but the per-channel precision allocation was not hardware-friendly. Jacob et al. (2017b) gave an integer arithmetic convolution for efficient inference, but it's hard to be used in training because the scale and bias of the quantized output tensor should be known before calculation. These post-training quantization methods provide enlightening theoretical analysis on quantization error, and show that CNN models with reduced bit-width still have adequate representation ability. However, their quantization process are based on pretrained models, which bring no acceleration in the training process.

2.2. Quantize-Aware Training

Quantize-aware training considered quantization effects in the training process. Some methods trained an ultra low-bit network like binary (Rastegari et al., 2016) or ternary (Li et al., 2016) networks, with a layer-wise scaling factor for a better representation ability. However, the extremely low bit-width led to notable performance degradation. Other methods sought to retain the accuracy with a relatively higher precision like 8-bit (Jacob et al., 2017a). Gysel et al. (2018) developed a GPU-based training framework to get dynamic fixed-point or Minifloat models. Choi et al. (2018) parameterized the clipping value in the activation function to properly restrict the range of activation. Dong et al. (2019) applied quantization during training based on the hessian spectrum. These methods can all retain an acceptable accuracy, however the training process is still using floating-point operations and running on GPUs.

2.3. Low-Bit Training

To accelerate the training process, some studies try to simplify floating-point format. A non-standard floating-point format is considered to be more suitable for data distribution in CNN (Dillon et al., 2017). Köster et al. (2017) proposed Flexpoint data format containing 16-bit mantissa and 5-bit tensor-shared exponent (sacle) to make training more efficient, which is similar to dynamic fixed-point format in (Gysel et al., 2018). Recently, 8-bit Floating-Point (FP8) (Wang et al., 2018; Mellempudi et al., 2019) were used with chunk-based accumulation and hybrid format to solve swamping. Some studies quantized both forward and backward process. Zhou et al. (2016) firstly proposed to use

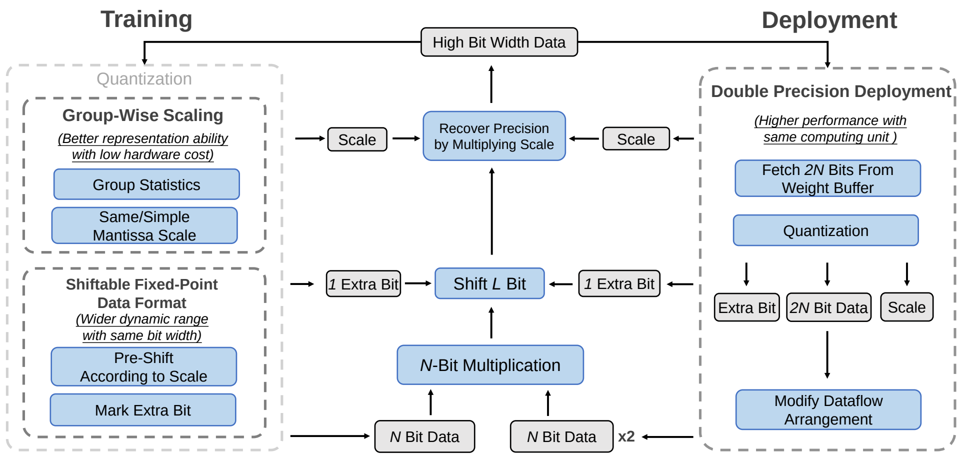

Figure 2. Overview of our methods. Float data is quantized by **group-wise scaling** to **shiftable fixed-point data format** before Conv, and **double width deployment** is used to utilize high bit-width weight buffer with the same computation unit.

the low-bit fixed-point gradient to accelerate both inference and training. Wu et al. (2018) implemented a full-integer training framework for integer-arithmetic machines. However, they caused notable accuracy degradation. Banner et al. (2018a) achieve a comparable accuracy with the full-precision baseline with hybird 8-bit and 16-bit quantization based on interger arithmetic in (Jacob et al., 2017b). But it's not very suitable for training as we discussed earlier.

3. Preliminary

3.1. Basic Quantization Method

In CNNs, floating-point numbers are always quantized with the help of auxiliary parameters that can increase the representation ability of fixed-point numbers. The relationship between a floating-point number and a fixed-point number is:

$$float \approx scale * (Fix + Bias) \tag{1}$$

In order to simplify the calculation, we choose unbiased quantization in our fixed-point training algorithm. As we only use quantized value for multiplication, unsigned quantization with extra sign bit is used. Therefore, the basic formula of our quantization method is:

$$Fix = Q(float) = Round(Clip(\frac{float \times 2^N}{scale}, 2^N)) \tag{2}$$

where $scale$ denotes the scaling factor, usually selected according to the maximum value of the float data, detailed methods to choose scale will be discussed in Section 4.1. And $\frac{\cdot \times 2^N}{scale}$ makes data distribution fit the representation range of the fixed-point number. $Clip(X, r)$ is the function that truncates the values of X to inside range $[-r, r]$. Moreover, $Round(\cdot)$ is the most fundamental part of the quantization method. Instead of using the simple nearest rounding, we adopt the stochastic rounding proposed by (Gupta et al., 2015):

$$Round(x) = \begin{cases} \lceil x \rceil & \text{w.p. } x - \lfloor x \rfloor \\ \lfloor x \rfloor & \text{w.p. } \lceil x \rceil - x \end{cases} \tag{3}$$

where "w.p." denotes "with probability".

3.2. Computation Flow for CNN

In this paper, we denote the filter coefficient and feature map of convolution as weight (W) and activation (A), respectively. In the back-propagation, the gradient of convolution results and weights are denoted as error (E) and gradient (G), respectively. As shown in Figure 1, generally, convolution is followed by batch normalization (BN), nonlinear activation (here, ReLU is used) and other operations (Ops) like pooling in a convolutional layer (Conv layer). Since convolution is the majority computational cost of the CNN training, we focus on the quantization before all three types of Conv(W, A) (Convolution of weight and activation), Conv(W, E), and Conv(A,E), as shown in Figure 1. And the output data of Conv is in floating-point format for other operations like BN.

4. Methods

Our fixed-point training framework applys quantization before Conv, and it consists of 3 techniques, as shown in Figure 2. First, we propose the **group-wise scaling** technique,

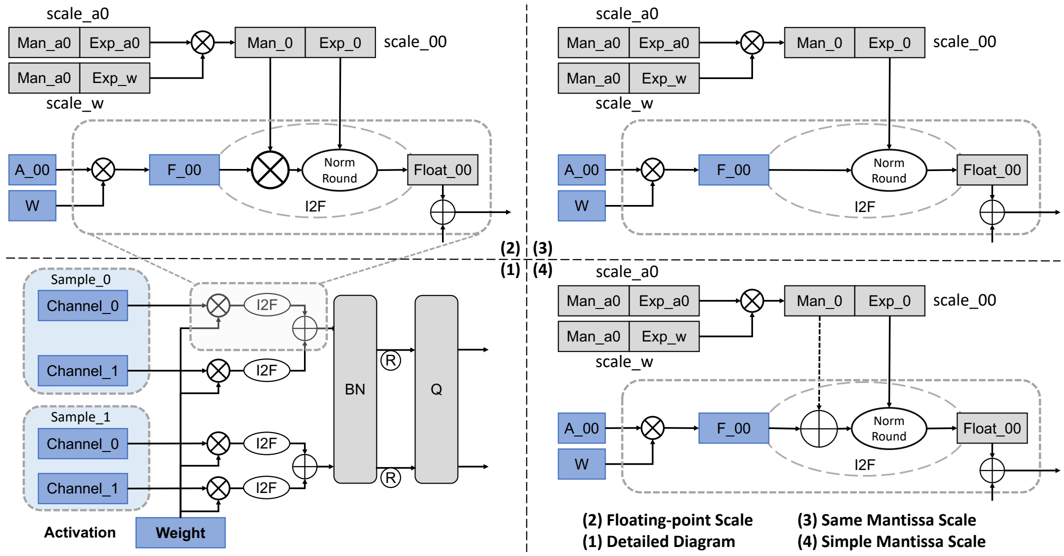

Figure 3. (1) Computation flow of Conv Layer in inference. Multiplication results are accumulated on the channel dimension, and convolution results are added by the BN on the sample dimension. (2) An expensive multiplication between Man_0 and F_00 is needed for group-wise floating-point scale. (3)(4) The computation will be simplified by "same mantissa scale" or "simple mantissa scale". "Norm Round" is a sub-operation of a float adder.

which calculates scaling factor of each group dynamically based on its maximum value. Then, scaled values is quantized to **shiftable fixed-point data format** with N-bit (the bit-width of the multiplications in convolutions) and one extra bit. Finally, we introduce the **double width deployment** for a better inference performance, the high-precision weight buffers are quantized to two N-bit fixed-point values instead of one. Only a simple dataflow arrangement is needed to implement double width deployment with the same N-bit fixed-point multiplier.

4.1. Group-wise Scaling

W, A, and E of each layer are 4-dimension tensors in training. For A and E, the four dimensions are sample in batch, channel, feature map height, and feature map width, respectively. Using a same scaling factor for all values in one tensor is a common practice to get quantized values, but it will lead to precision loss for most of values. The blue line in Figure 4 shows the max value in each group when the A and E are grouped by channel or sample ($group\_max$). If we use the overall maximum value ($tensor\_max$, green lines in Figure 4), many values will waste their precision because most of the groups have no data close to the overall maximum. There are even about half of the groups, in which all data is smaller than $0.5 \times tensor\_max$ (red line).

Using group-wise scaling factors is a promising direction to reduce the loss, such as splitting a tensor to n groups by sample or c groups by channel and make scaling factors

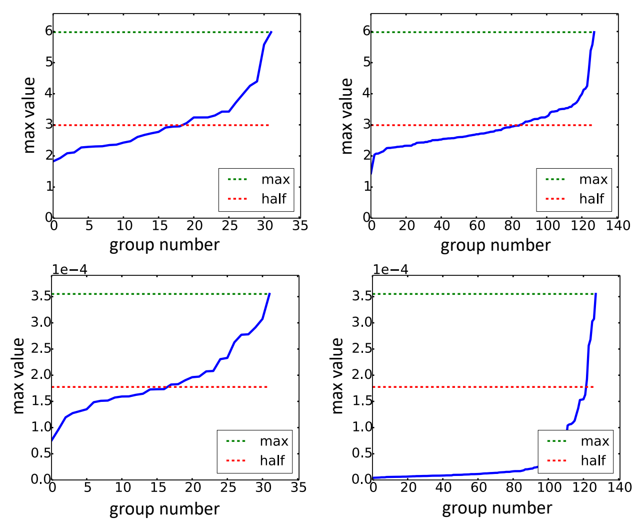

Figure 4. Maximum value of each group of A(above) and E(below) , grouped by channel(left) or sample(right).

shared across values in one group. **Group-wise floating-point scale** is simple and intuitive when doing the quantization, however, it will introduce overhead in Conv. The relationship of the float number and the quantized fixed-point number is:

$$float_i = scale_i \times Fix_i \tag{4}$$

where i is the group index, and values in the same group

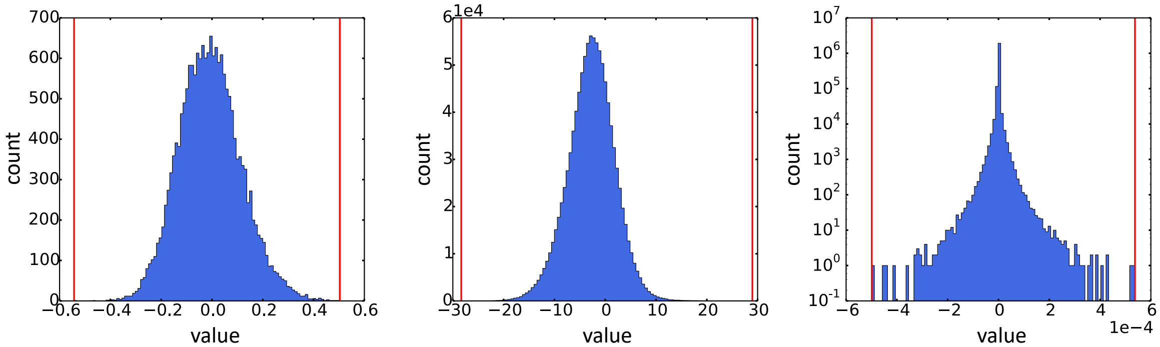

Figure 5. Distribution of W, A, E of one layer during the training of ResNet-20 on CIFAR-10.

share the same $scale_i$. We can assume that $scale$ has a floating-point format, which is composed of the mantissa and exponent. Then Equation 4 can be written as follows:

$$float_i = Man_i \times 2^{Exp_i} \times Fix_i \tag{5}$$

It should be pointed out that the standard convolution needs to accumulate multiplication results on the channel dimension, and the BN sums convolution results on the sample dimension, as shown in Figure 3(1). So, when A in Figure 3 is grouped by channel, inner results Fix of different channels will have different $scale$ values. If we want to get the real result of $float_{00} + float_{01}$, we can't simply calculate $Fix_{00} + Fix_{01}$ and have to calculate it as:

$$\begin{aligned} float_{00} + float_{01} = &(Fix_{00} \times Man_0) \times 2^{Exp_0} \\ &+ (Fix_{01} \times Man_1) \times 2^{Exp_1} \end{aligned} \tag{6}$$

where we consider $(Fix_{00} \times Man_0) \times 2^{Exp_0}$ as a new floating-point number and do the floating-point addition. This is an expensive Integer to Float (I2F) operation, because $(Fix \times Man)$ is an expensive multiplication that can't be simplified on the hardware, as shown in Figure 3 (2). So, we propose the "same mantissa scale" and "simple mantissa scale" methods. By applying a simple restriction on scaling factor selection , we can reduce the overhead in Conv.

The **"same mantissa scale"** means that we select the smallest value that is no less than $group\_max$ in the set $\{tensor\_max \times 2^K | K \in \mathbf{Z}\}$ as the scale of each group. These scales are 2^k times of each other, that is to say, they have the same mantissa. For example, $Man_0 = Man_1 = Mantissa$ in Equation 6, so the calculation is simplified to:

$$\begin{aligned} float_{00} + float_{01} = &Mantissa \times \\ &(Fix_{00} \times 2^{Exp_0} + Fix_{01} \times 2^{Exp_1}) \end{aligned} \tag{7}$$

where $Mantissa$ is a tensor-wise scale and doesn't participate in I2F, so the expensive multiplication is not needed, as illustrated in Figure 3(3).

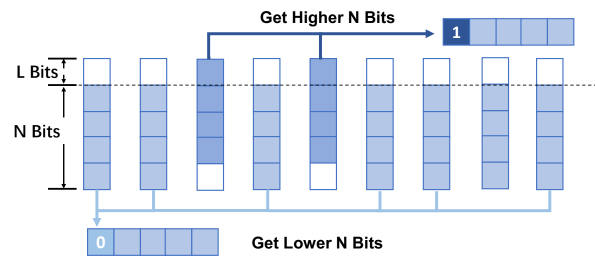

Figure 6. Quantization to the shiftable fixed-point data format.

The **"simple mantissa scale"** is to use different but simple scales. Choosing the powers of two as scales is the simplest solution. Besides, we can choose a slightly more complicated one by limiting Man to 2-bit. That means selecting the smallest value that is no more than $group\_max$ in the set of $\{2^K, 1.5 \times 2^K | K \in \mathbf{Z}\}$. In this way, Equation 6 remain the same but the calculation of $(Fix_{00} \times Man_0)$ is simplified to an addition , as shown in Figure 3(4).

4.2. Shiftable Fixed-Point Data Format

One of the significant challenges in quantization is to balance the overflow error and the rounding error. (Han et al., 2015) pointed out that numbers with larger magnitudes play a more important role. However, if the scaling factor is chosen to minimize the overflow error, the rounding error will increase for all values. On the other hand, if a smaller scaling factor is used, large values will be truncated due to the limitation of the representation range, resulting in considerable performance degradation.

Figure 5 shows the typical distribution of W, A, and E of a convolution layer in the training process. The red line indicates the extreme values. We can find that W and A are approximately Gaussian distributed, and a large number of E are concentrated near zero, while others are approximately

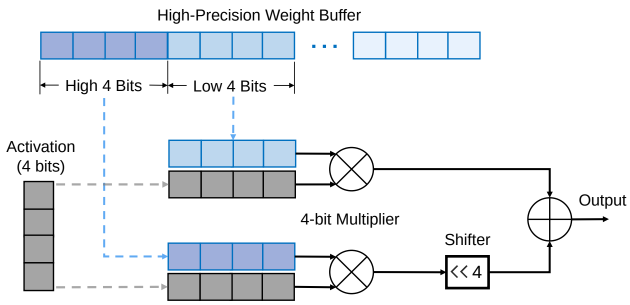

Figure 7. Illustration for double width deployment

distributed following the Laplace distribution. As shown in Figure 5, a large number W/A/E are smaller than half or even one-quarter of the max value. In consequence, if the common fixed-point format is adopted and the scaling factor is chosen according to the max value to eliminate the overflow error, the highest one or two bits of many quantized data will be zero. That is to say, the high-order bits are not effectively used.

To better utilize the representation ability of all bits, we propose a **shiftable fixed-point data format**. As illustrated in Figure 6, the values smaller than $2^{(N-L)}$ after group-wise scaling are left shifted L bits (multiply 2^L), and an extra bit denotes whether the value is shifted or not. Then, stochastic rounding is applied to get N-bit numbers. The 2^{-L} is called the **shifting scale**, and L is referred to as **shifting bit**. Now we preset L for W/A/E in our algorithm. Compared with the common fixed-point format, L more effective bits are used for representing the values that are smaller than $2^{(N-L)}$, without increasing the overflow error. Therefore, the proposed shiftable fixed-point data format has better representation ability.

Note that although an extra bit is needed to mark whether to shift, this bit does not participate in the multiplication. In consequence, the N-bit fixed-point multiplier does not need to be changed to handle this data format.

4.3. double width deployment

The magnitudes of the updates are much smaller compared to those of the weights in CNN training. Therefore, same with other work, there is a **high-precision buffer** in our low-bit training framework, in order to keep gradients from swamping (being rounded to zero because they are much smaller than other weights). As shown in Figure 1, we get N-bit quantized weight from the higher-precision weight buffer. The usual scheme is to take the same bit-width in inference as in training, but we find that directly using the high-precision buffer for inference outperforms using the reduced-precision version. That is to say, the low-bit training framework can obtain an effective floating-point model.

Based on this discovery, we proposed **double width deployment**. Instead of fetching N bits identical to the training stage, we fetch $2N$ bits and split them in half for computation, which exploits the high-precision weight buffer. The amount of calculation is equivalent to doubling the number of channels and we can boost the performance by only modifying the data flow. As shown in Figure 7, during the inference process, one multiplication is extended to two multiplications and one addition. The double width deployment does not introduce extra computing elements and it is still compatible with the hardware used for low-bit training.

Noted that the double precision extension is only applied in the deployment stage, while the training stage remains in using N-bit.

5. Experiments

5.1. Experiment Setup

We apply the low-bit fixed-point training method to variants of ResNet (He et al., 2016) on CIFAR-10 (Krizhevsky, 2010) and ImageNet (Deng et al., 2009). For all the floating-point and fixed-point training experiments, we use the standard data augmentation and hyper-parameters following (He et al., 2016). For both datasets, SGD with momentum 0.9 and weight decay 1e-4 is used, and the initial learning rate is set to 0.1. We train the models for 90 epochs on ImageNet, and decay the learning rate by 10 every 30 epochs. For training on CIFAR-10, we train the models for 160 epochs and decay the learning rate by 10 at epochs 80 and 120.

5.2. Results on CIFAR-10 and ImageNet

We apply our method to train ResNet-20 on the CIFAR-10 dataset with different quantization bit-width, and Table 2 shows the results of various methods. Generally, the E values are known to be sensitive to quantization and cannot be quantized with a low bit-width (Zhou et al., 2016). Interestingly, our method can quantize E with a bit-width as low as 2, with a small accuracy drop from 92.2% to 90.75%.

For the evaluation on a large dataset, we apply our method to training ResNet-18 on the ImageNet dataset. Table 3 shows the comparison of our method and other fixed-point or low-bit training methods. The accuracy degradation of our method is very small compared with the floating-point model under 8-bit quantization (0.6% accuracy drop from 69.7% to 69.1%). In the cases with lower bit-width, our method demonstrates a better performance than all the baseline methods. More specifically, with only 6-bit, our method achieves a higher accuracy (68.2%) than the 8-bit results (66.4%) of RangeBN (Banner et al., 2018a). Moreover, since we adopt the same quantization bit-width in the for-

Table 2. Fixed-point training results on CIFAR-10, when W/A/E are splited to n/c/n groups and their L are 1/1/2. "f" indicates that floating-point representation is used, and "-double"means testing with our double width deployment.

Method	Bit-Width (W/A/E)	Model	Accuracy	Baseline
TBP (Banner et al., 2018a)	1 1 2	ResNet-18	81.5%	90.36%
WAGE (Wu et al., 2018)	2 8 8	VGG-like	93.2%	94.1%
XNOR-Net (Rastegari et al., 2016)	1 1 32f	ConvNet	89.83%	91.8%
Ours	4 4 4	ResNet-20	91.46%	92.2%
Ours	3 3 3	ResNet-20	89.12%	92.2%
Ours-Double	3 3 3	ResNet-20	91.74%	92.2%
Ours-Double	2 2 2	ResNet-20	90.75%	92.2%

Table 3. Fixed-point training results on ImageNet, when W/A/E are splited to Cin/n·C/n·C groups and their L are 1/1/1. "f" indicates that floating-point representation is used, and "-double"means testing with our double width deployment.

Method	Bit-Width (W/A/E)	Model	Accuracy	Baseline
DoReFa (Zhou et al., 2016)	8 8 8	AlexNet	53.0%	55.9%
WAGE (Wu et al., 2018)	2 8 8	AlexNet	48.4%	56.0%
RangeBN (Banner et al., 2018a)	8 8 16	ResNet-18	66.4%	67.0%
HFP8 (Mellempudi et al., 2019)	8f 8f 8f	ResNet-18	69.0%	69.3%
Ours	8 8 8	ResNet-18	69.1%	69.7%
Ours	6 6 6	ResNet-18	67.4%	69.7%
Ours	4 4 4	ResNet-18	63.2%	69.7%
Ours-Double	6 6 6	ResNet-18	68.2%	69.7%
Ours-Double	4 4 4	ResNet-18	64.9%	69.7%

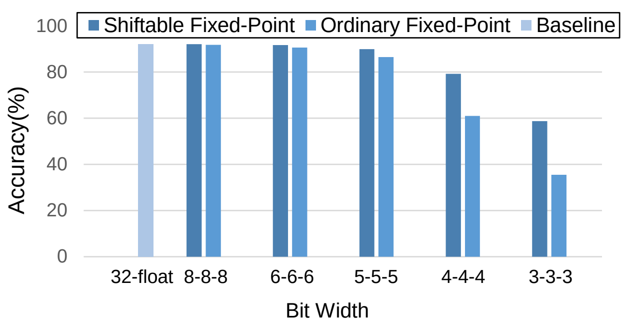

Figure 8. The results of ordinary fixed-point training and training with shiftable fixed-point data format, no grouping for both.

ward and backward process, the hardware design could be simplified.

5.3. Ablation Studies

5.3.1. Shiftable Fixed-Point Data Format

To demonstrate the effectiveness of the shiftable data format, we compare the accuracy of fixed-point training with the shiftable fixed-point data format and the ordinary fixed-point data format. As shown in Figure 8, training using the shiftable fixed-point data format achieves better performance across different bit-width choices, especially in the cases with an extremely low bit-width (e.g. 3-bit, 4-bit).

We conduct further exploration to see whether different shifting scales should be used for different tensors. From the experimental results shown in Table 5, one can see that the most appropriate shifting scale of error (E) in training is $0.125\times$ (shifting bit $L = 3$), while the shifting scales of weight (W) and activation (A) are usually $0.5\times$. This observation is consistent with the distribution of E shown in Figure 5, in which a large amount of the error values are concentrated around 0 while a few values with very large magnitudes exist. A longer shifting (e.g., 3-bit for $0.125\times$) handles extremely distributed values better, and takes both the overflow error of the large values and the rounding error of smaller values into consideration.

5.3.2. Group-wise Scaling

As discussed in Section 4.1, independent group-wise high-precision scaling factors could not be used in hardware-friendly quantization. The simplest way is to use the power of 2 as the group-wise scales. Besides this simple solution, we propose another two group-wise scaling methods: "same mantissa scale" and "simple mantissa scale". As shown in Table 4, the group-wise scaling methods could achieve significantly better results than using the floating-point scale in the cases of low bit-width (e.g., above 90.0 % V.S. 78.95 %). Among these group-wise scaling methods, "simple mantissa scale" group-wise scaling is the best.

When the group-wise scaling methods are applied with the

Table 4. The accuracies of using different scaling methods with different bit-width choices, and the shiftable data format is not applied. "'Float Scale" means floating-point scales without grouping and "Power Scale" is a simplified case of same mantissa scale as discussed in Section 4.1.

Bit-Width	Float Scale	Group-wise Power Scale	Group-wise Same Mantissa Scale	Group-wise Simple Mantissa Scale
6 6 6	91.26%	**91.71%**	91.55%	91.67%
5 5 5	88.87%	91.40%	91.35%	**91.68%**
4 4 4	78.95%	90.08%	90.54%	**90.66%**
3 3 3	62.50%	80.06%	82.73%	**84.29%**

Table 5. The results of using different shifting bit L for weight (W), activation (A), and error (E), when their N is 4/4/4 and there is no grouping.

Shifting Bit L (W/A/E)	Accuracy
1 1 1	88.38%
2 1 1	88.56%
1 2 1	87.68%
1 1 2	88.92%
1 1 3	89.53%
2 1 3	**90.63%**

shiftable fixed-point data format, the performance is further boosted. From the results shown in Figure 9, we can see that the fixed-point training results of the combination of the two methods are consistently better than the results of only using one method across different bit-width choices.

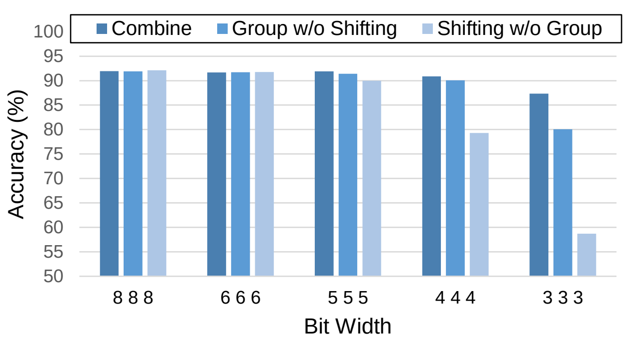

Figure 9. The results of using shiftable fixed-point data format without grouping, using ordinary fixed-point data format and group-wise scaling, and the combination of the two methods.

5.3.3. DOUBLE WIDTH DEPLOYMENT

The results of double width deployment are shown in Table 6. We can see that the double width deployment can improve the test accuracy of the model without any additional training. Doubling the deployment precision of W and A alone could improve the accuracy, and doubling W is relatively more effective. Moreover, doubling both W and A leads to a larger improvement. It is worthwhile to note that, the double width deployment of the model trained with 2-bit fixed-point multiplications could achieve an accuracy of 90.75%. The last row in Table 6 shows the double-precision deployment results of the 2-bit trained model after 2 epochs of double-precision finetuning. With a small overhead of finetuning, the accuracy could be further restored to 91.1% (close to the floating-point accuracy 92.2%).

Table 6. The accuracies are improved by double width deployment, when models are trained with our low-bit fixed-point training framework with both group-wise scaling and shiftable data format.

Bit-Width	Finetune	None	W	A	W+A
4 4 4	No	91.46	-	-	-
3 3 3	No	89.12	91.41	90.12	**91.74**
2 2 2	No	87.34	89.14	88.72	**90.75**
2 2 2	Yes	-	89.34	89.18	**91.1**

5.4. Energy consumption estimation

Our method simplifies the convolution multiplications from single-precision floating-point to low-bit fixed-point, which account for the main computational cost of CNN training, as shown in Table 1. While our method could significantly reduce the cost of the convolution operations, it also introduces some overhead:

1) Calculating group-wise scales, shift-and-intercept operations are needed in our quantization scheme. The cost of these two types of operations is similar to floating-point multiplications and additions, and the number of these operations is the same as that of BN and weight updating.

2) For handling the shiftable fixed-point data format, multiplexers are required in Conv. We ignore this part of energy consumption in this estimation. Multiplexers with 5 inputs will support all L no more than 2, so online adjustment of L within this range will not bring additional overhead.

3) Our group-wise scaling methods brought I2F in Conv. When using the "same mantissa scale" method, the I2F conversion could be done via a simple shift operation (Norm and Round). When using the "simple mantissa scale" method, the I2F conversion is done via a fixed-point addition. To reduce the computational cost of this additional add operation,

we do not group the "simple mantissa scale" of A and E by channels, but only by samples. In this way, the number of extra add operations could be reduced to the same number of batch normalization operations.

To summarize, the introduced overhead is small compared with the reduced cost. According to the cost of single fixed-point and floating-point operation in Table 7, our algorithm has the potential to save at least 75% energy cost of the computation cost when we train ResNet-18 with $N = 8$. Actually the internal memory cost will also be alleviated when a lower bit-width representation of W/A/E.

Table 7. The cost estimation of primitive operations with 45nm process and 0.9V. (Sze et al., 2017)

Params	Energy(pJ)		Area(μm^2)	
	Mul	Add	Mul	Add
8-bit Fix	0.2	0.03	282	36
16-bit Float	1.1	0.40	1640	1360
32-bit Float	3.7	0.90	7700	4184

6. Discussion

At present, the hyperparameters of the two technologies: grouping method and shifting bit L are preset according to the empirical observation, and they are the same in the whole network, which can not guarantee the optimality. It will improve the availability of the algorithm if we automatically determine these hyperparameters according to the data distribution at the beginning of training, and this is left for future work.

The hardware implementation and evaluation can help to prove the value of this work. We roughly estimate the computation energy consumption for a qualitative explanation with the count of Ops in Table 1 and the cost of Ops in Table 7 in Section 5.4. The detailed hardware design will be another future work.

7. Conclusion

In this paper, we propose a fixed-point training framework composed of 3 techniques. We strike a balance between rounding error and overflow error in quantization with the shiftable fixed-point format, enrich low-bit data's representation ability via group-wise scaling, and boost the performance with the same hardware multiplier through double-precision deployment. Using these three techniques, the ResNet-20 model acquires an accuracy of 91.41% (0.79% accuracy drop) on CIFAR-10 with only 3-bit multiplication. On ImageNet, the ResNet-18 models trained with 6-bit and 8-bit multiplications achieve accuracies of 68.2% and 69.1%, respectively, which is better than that of the state-of-art fixed-point training method (66.40%).

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